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Structural Performance of Castellated Steel Beams with Flange-Connected Ring Stiffeners and Inverted Stiffener Geometry

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ABSTRACT

This paper aims to study the behavior of castellated beams with large openings (D/H = 0.7) with a constant offset of 10 mm. Different configurations, including ring stiffeners connected to the flanges and geometrically inverted ring stiffeners, were investigated. Nonlinear finite element models in ABAQUS were used to compare circular and hexagonal openings. The hexagonal openings with flange-connected showed an increase in the ultimate load by up to 6.12% and improved the ductility factor from 5.65 to 6.33. Connecting the ring stiffeners of the circular openings to the flanges slightly reduced the ultimate load by up to 1.67%, while the ductility factor decreased from 4.25 to 3.39. The inverted geometrical cases led to lower performance, especially when the hexagonal openings were strengthened with a circular stiffener, with a decrease of 11.36%. Ring stiffeners connected to the flanges of castellated beams with large hexagonal openings can be used to control the web post buckling.

الأداء الإنشائي للعوارض الفولاذية المسننة ذات حلقات التقوية المتصلة بالشفة وهندسة التقوية المعكوسة

محمد العربي منير [1,*]، عثمان أبوبكر الحضيري [1]، إبراهيم عمر الترهوني [1]

الكلمات المفتاحية

الكمرات المسننة
الكمرات الخليوية
الأباكوس
المقويات الحلقية
تحليل العناصر المحدودة

الملخص

تهدف هذه الورقة إلى دراسة سلوك العوارض المسننة ذات الفتحات الكبيرة (D/H = 0.7) مع إزاحة ثابتة قدرها 10 مم. تمت دراسة تكوينات مختلفة، بما في ذلك الدعامات الحلقية المتصلة بالشفاه والدعامات الحلقية المعكوسة هندسياً. تم استخدام نماذج العناصر المحدودة غير الخطية في ABAQUS لمقارنة الفتحات الدائرية والسداسية. أظهرت الفتحات السداسية المتصلة بالشفة زيادة في الحمل الأقصى بنسبة تصل إلى 6.12% وتحسناً في معامل اللدونة من 5.65 إلى 6.33. ربط الدعامات الحلقية للفتحات الدائرية بالشفاه قلل قليلاً من الحمل الأقصى بنسبة تصل إلى 1.67%، بينما انخفض معامل اللدونة من 4.25 إلى 3.39. أدت الحالات المقلوبة هندسيا إلى أداء أقل، خاصة عند تقوية الفتحات السداسية بمقوى دائري، مع انخفاض بنسبة 11.36%. يمكن استخدام الدعامات الحلقية المتصلة بشفاه العوارض المسننة ذات الفتحات السداسية الكبيرة للتحكم في انبعاج الجذع.

Introduction

Castellated and cellular beams have been used in many modern construction projects due to their several advantages such as strength-to-weight ratio and capability to pass service ducts and pips through the web opening. However, these advantages come with undesirable defects, whereby the reduction of web materials mainly changes the stress distributions of the beam and creates a failure mechanism absent in the solid beam. Consequently, accurate geometrical design and strengthening strategies must be adopted to address such defects.

The manufacturing process of castellated beams involves cutting a standard I-section web in a zigzag or hexagonal pattern and reconnecting the two halves, thus increasing the total length of the section by about 50 % without adding any additional weight to the beam itself [1]. Although this increases the section modulus and flexural rigidity, the resulting openings in the web give rise to a number of critical failures.

Kerdal and Nethercot (1984) [1] were the first researchers to study these failure modes and identify them in several ways, such as the Vierendeel mechanism, web-post buckling (WPB), lateral-torsional buckling (LTB), and welded joint rupture. Their results showed that the Vierendeel mechanism was dominant in most cases of failure modes. The web, post failure, as observed by [2], becomes increasingly significant as the depth and spacing of the opening increases, especially when the depth of the opening reaches about 80 % of the section height. At this point, the stiffness has significantly deteriorated and the stress concentration at the insertion angles significantly increased.

Numerous authors [3] have experimentally investigated web opening configuration. The findings show that a hexagonal opening with a ratio of 0.56 measurably affects performance,

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making improvements of up to 50% compared to solid beams. The results serve as a practical and dependable reference, validating design choice as an optimal solution. [4] conducted a comparative study on various opening geometries, demonstrating that beams with diamond-shaped openings show improved shear resistance. The improvement can be attributed to their larger cross-sectional area, which facilitates better load transfer. Furthermore, a study by [5], employing a numerical model in Abaqus software, found that circular openings achieve an optimal balance between linear displacement and rotational resistance. [6] observed, through finite element analysis with the ANYSYS program, that beams with circular openings experience greater deflection than beams with hexagonal openings of equivalent area, thus confirming the impact of the opening shape on the structural performance of the beams.

Many stiffening configurations have been studied by researchers to compensate for the loss of strength and rigidity resulting from the trunk openings. [7] showed that deflection can be reduced and the load capacity of beams increased by using end supports at the beam ends, while [8] demonstrated through laboratory tests on beams with large rectangular openings that the use of horizontal stiffeners effectively increases the beam’s load capacity. [9] examined various stiffener configurations, including horizontal, vertical, sleeve, and boundary stiffeners, across sections with varying compactness. Their results showed that the use of vertical and boundary stiffeners led to an increase of up to 50% in the load capacity of slender sections. They observed that the common failure mode shifted from web- column buckling to bending failure with the use of vertical and boundary stiffeners. In contrast, horizontal stiffeners had little effect on the failure mode. Recently, ring stiffeners have emerged as an effective method for strengthening beams with openings, especially octagonal and circular ones. [10] explained that the use of circular welded ring stiffeners around openings increased the load capacity by 188%, while the increase was 77.6% with octagonal ring stiffeners. In 2025, a numerical study conducted by [11] using the Abaqus program, offsetting the rings by 10 mm from the edge of the opening, instead of traditionally placing them directly on the edge, effectively reduced local stress concentrations and delayed the onset of buckling in beams with hexagonal openings with 0.7 D/H.

In addition to steel strengthening, several recent studies have demonstrated the use of carbon fiber-reinforced polymer (CFRP) as a lightweight and effective alternative. Jadhav et al. (2024) [12] found that reinforcing beams with diamond-shaped openings using CFRP strips achieved the highest ultimate load capacity compared to other studied configurations. [13] showed that beams reinforced with transversely- oriented CFRP bracing outperformed beams attached directly around the openings in preventing stem buckling.

Finite element modeling, particularly using ANSYS software and Abaqus, has been the methodological basis for most recent research, with accuracy in predicting deflection reaching only 1.9% variance. [14]. [15] further investigated the value of preliminary eigenvalue buckling analysis as a precursor to a nonlinear static analysis to accurately predict failure load.

In general, most previous studies agreed that the castellated steel beams have a clear structural effectiveness, but they are highly sensitive to the shape, size, spacing, and location of the web openings. Current literature also agreed that using stiffeners to strengthen the castellated beams can significantly improve load capacity, reduce deflection, and delay local instability. However, previous studies differed in their focus and conclusions. Some studies focused directly on the shape and geometry of the openings, such as circular, hexagonal, diamond, rectangular, sinusoidal, or octagonal openings. Others examined the stiffener type (circular, vertical, boundary or horizontal) as well as CFRP stiffeners. In general, the results of these studies have shown that there is no specific shape of openings or stiffener layout that is always the best; rather, it depends on several conditions such as the ratio of openings, loading condition and the failure mode to be controlled.

Despite significant contributions to this field, a considerable research gap remains, which requires more investigation. So far, only a few studies have focused on ring stiffeners directly connected to beam flanges. Moreover, the structural performance of castellated steel beams with geometrical inverted ring configurations, such as circular opening with hexagonal stiffeners or vice versa, has not been well examined. This specific gap needs more comprehensive investigation, as these innovative configurations may influence load transfer paths between the flanges, rings, and web, either enhancing or compromising the structural response governed by the opening geometry.

Methodology

Model Configurations

A parametric analysis examined beams featuring both hexagonal and circular openings. For each opening type, two stiffener configurations were analyzed: one with a consistent stiffener shape and another with an inverted configuration. (i.e. hexagonal openings were reinforced with both hexagonal and circular ring stiffeners, while circular openings received the same treatment). Additional models were created to investigate whether extending the ring stiffener to the flanges enhanced load transmission and delayed local failure. The different model configurations considered in this study are illustrated in Figure 1 and materials properties used in FE model are described in Table 1. After completing the simulations, the primary findings were identified and compared across all configurations.

Table 1:: Steel Material properties

Material properties	Value
Young's modulus *E*	210 GPa
yield stress σ_y	250 MPa
Ultimate tensile strength σ_u	410 MPa
Poisson's ratio *v*	0.3
Strain hardening modulus *ET*	5000 MPa

Finite Element Modelling

In this study, a three-dimensional FE model previously developed by Munir and Alhodiry has been used to simulate the initial boundary conditions, material properties, and loading conditions for castellated beams [11]. The numerical model was designed to investigate the effect of the opening shape with inverted stiffeners as well as that of connecting the ring stiffeners to the flanges.

In the proposed model, the main parts of the castellated steel beams such as main beam, ring stiffeners, connected stiffeners and loading plate stiffeners were modeled by the three-dimensional solid element, C3D8R. The element contained eight- node, reduced-integration brick elements

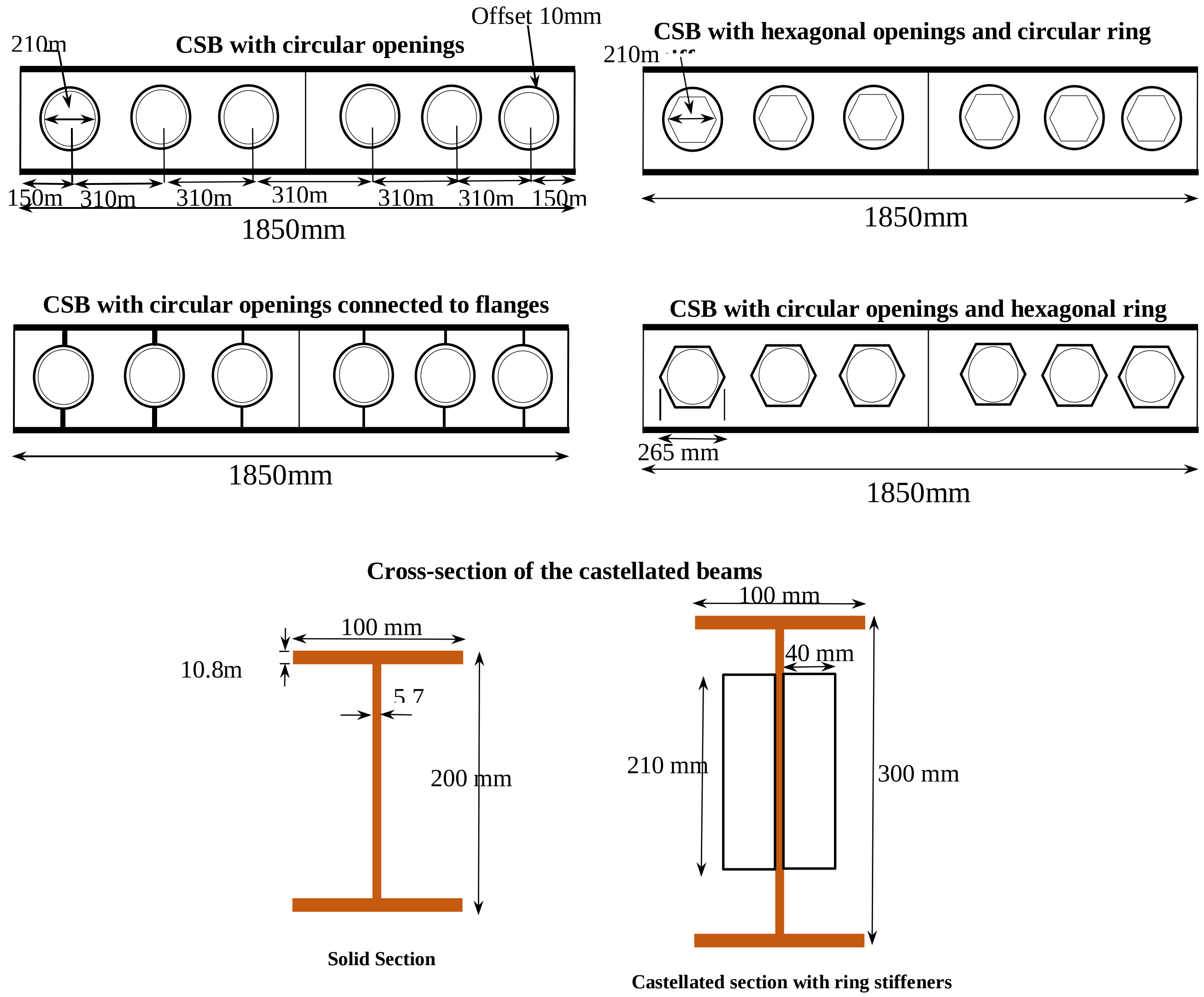

Figure 1: Geometry and Stiffener Configurations of the Proposed Castellated Beam Models

with three transitional degrees of freedom at each node (x, y and z). Material and geometric nonlinearities were incorporated in the finite element simulation. The Newton–Raphson technique was employed for the analyses. The isotropic properties of the material were assumed to simulate the actual behavior of castellated steel beams.

The interaction between the connected parts were modelled using tie constraints, with the beam surface assumed as the master surface and the loading plates and stiffeners a slave surface. This technique is used to ensure complete contact and to prevent relative slip between the components. Figure (2) shows the model's simply supported boundary conditions after analysis. One end of the beam is restrained in all translational directions, while the other end is allowed to act as a roller support A vertical load was applied at the mid-span of the upper flange using a displacement-controlled procedure to allow complete non-linear response of the beam until failure.

A refined mesh was generated around the web openings, stiffeners, and loading areas where high stress gradients and local instability were expected. A slightly coarser mesh was adopted outside these critical zones to reduce computational effort while maintaining acceptable accuracy. The selection of this mesh density was primarily adopted during the initial validation phase against experimental data. The selected mesh size was sufficient to capture local stress development around the openings and the interaction between the ring stiffener and the beam web as shown in Figure 2.

Figure 2: Finite element model with mesh and boundary conditions

Model validation

The proposed finite element model was primarily calibrated based on the experimental work reported in the literature [14], and previously presented by [11]. Figures (3 and 4) compare the results of the proposed numerical model and the laboratory experiments by [14]. The results demonstrated a good match between the curves, confirming the reliability of the suggested numerical model and facilitating a more comprehensive and targeted parametric investigation.

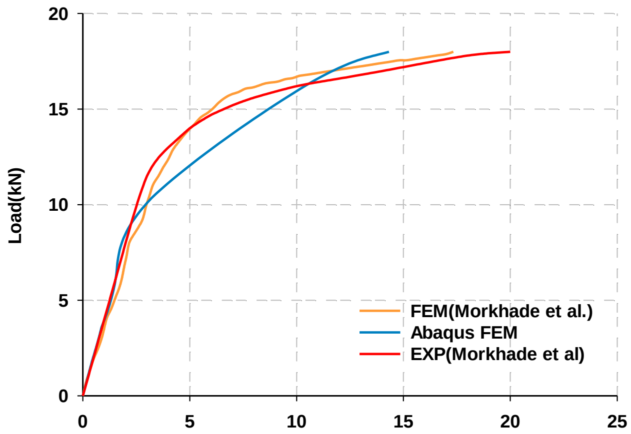

Figure 3: Verification curve of CSB1

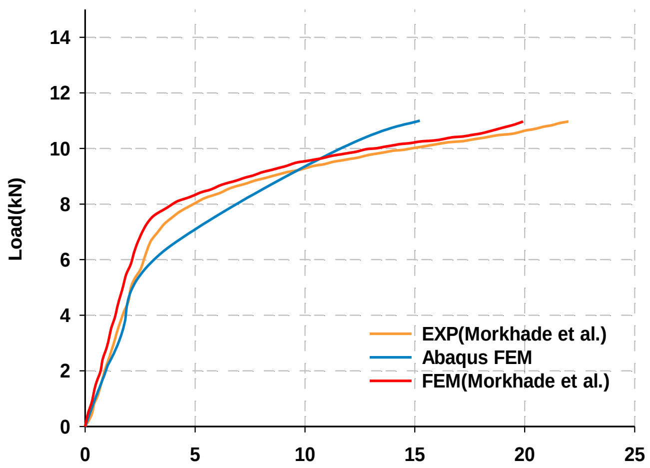

Figure 4: Verification curve of CSB2

Results and Discussion

The performance of beams with circular and hexagonal openings was investigated for maximum load capacity, displacement characteristics, and ductility. Table 2 illustrates the ultimate load and maximum displacement values for several beam configurations. The findings obtained through the numerical models demonstrated the influence of the inverted stiffeners and the flange-connected stiffeners on the stress distribution and structural performance of castellated steel beams, including hexagonal and circular openings, at a ratio of (D/H = 0.7). Figures (5-8) illustrate the relationship between load and mid-span deflection for all examined beams with different configurations. Figure (10) shows the Von Mises stress distribution for investigated beam configurations.

The opening shape's effect on Load capacity

Beams with hexagonal openings usually have more load-carrying capacity than those with circular openings under the same conditions. As shown in Table 2, the hexagonal beam (HEX-DH0.7-O-10) has an ultimate load of 272.45 kN as compared to 249.46 kN (CIR-DH0.7-O-10). Similarly, beams with a hexagonal opening demonstrated a higher ductility factor (5.65) than circular openings with a ductility factor of (4.25).

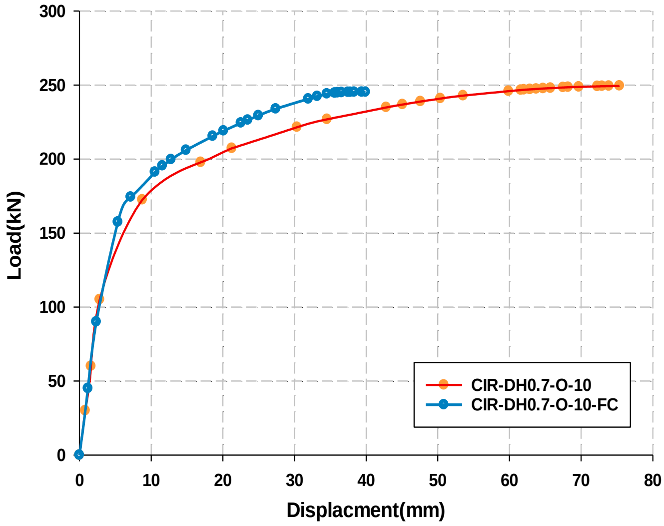

Figure 5: Load vs displacement of circular opening flange-connected with 0.7 D/H ratio

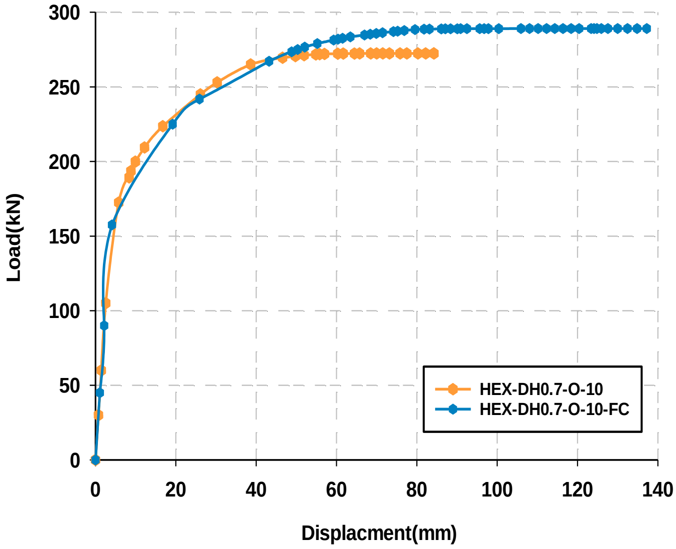

Figure 6: Load vs displacement of hexagonal opening flange-connected with 0.7 D/H ratio

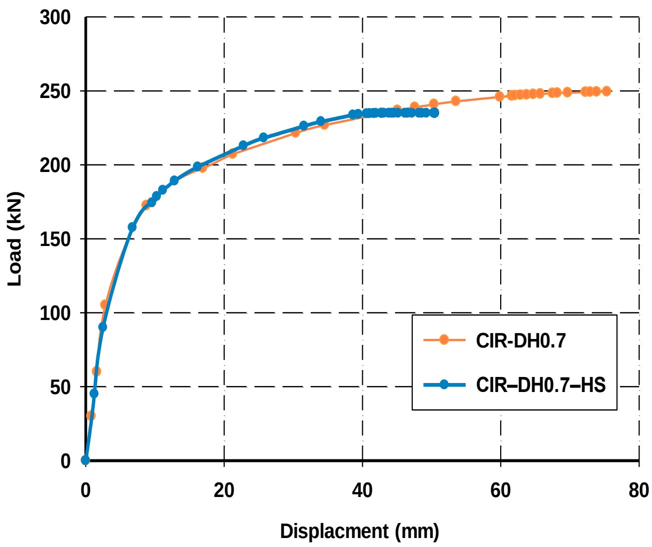

Figure 7: Load vs displacement of circular opening with 0.7 D/H ratio and hexagonal ring stiffener

Table 2: Results obtained from FE analysis for different configurations

S N o	Beam Type	Opening Shape	D/H Ratio	Offset distance (mm)	Beam configuration	Ultimate Load (kN)	Ultimate load difference (%)	Max disp. Δu (mm)	Yield disp. Δy (mm)	Ductility factor μΔ
1	**CIR-DH0.7-O-10**	Circular	0.7	10	-	249.46	0.00%	75.35	17.71	4.25
2	**CIR-DH0.7-O-10-FC**	Circular	0.7	10	Flange connected	245.29	-1.67%	39.90	11.79	3.39
3	**CIR-DH0.7-O-10-HS**	Circular	0.7	10	Hexagonal stiffener	234.96	-5.81%	50.49	12.51	4.03
4	**HEX-DH0.7-O-10**	Hexagonal	0.7	10	-	272.45	0.00%	84.24	15	5.65
5	**HEX-DH0.7-O-10-FC**	Hexagonal	0.7	10	Flange connected	289.11	6.12%	137.25	21.68	6.33
6	**HEX-DH0.7-O-10-CS**	Hexagonal	0.7	10	Circular stiffener	241.50	-11.36%	35.94	8.95	4.02

CIR = Circular, HEX = Hexagonal, DH=Depth-to Diameter ratio, O= Offset distance
FC=Flange connected , HS= Hexagonal stiffener, CS=Circular stiffener

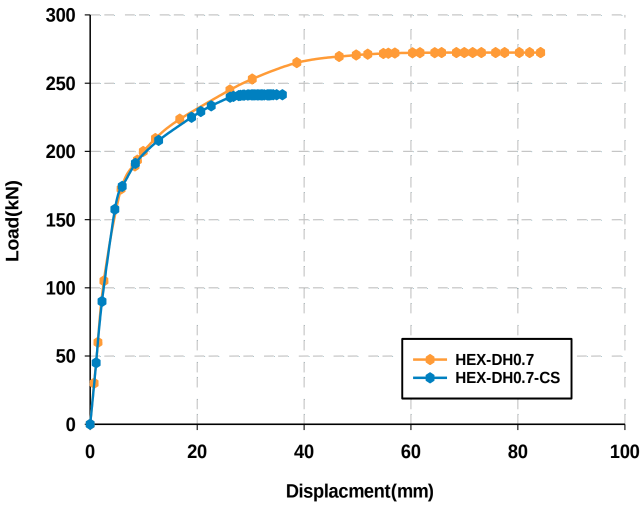

Figure 8: Load vs displacement of hexagonal opening with 0.7 D/H ratio and circular ring stiffener

The effect of the flange-connected (FC) configuration

The flange-connected (FC) modification showed shape-dependent effects on the beam's structural response. Of the hexagonal opening beam configurations, (HEX-DH0.7-O-10-FC) was the most beneficial of all models evaluated. The ultimate load increased to 289.11 kN, an increase of 6.12% percent. Moreover, this configuration significantly improved the deformation capacity, with a maximum deformation of 137.25 mm and a ductility factor of 6.33.

In contrast, the same modification had a less -beneficial effect on the circular-opening beam. For the model (CIR-DH0.7-O-10-FC), the ultimate load decreased slightly to 245.29 kN, representing a 1.67% reduction. More notably, the maximum displacement dropped substantially to 39.90 mm, leading to a reduced ductility factor of 3.39. The results demonstrate that the efficacy of a flange-connected (FC) configuration is significantly influenced by the opening shape, providing considerable improvement in hexagonal beams while reducing the deformation and ductility performance of circular beams.

The effect of inverted hexagonal and circular stiffener geometry

The shape of inverted stiffeners often resulted in a reduction in ultimate load capacity. The ultimate load for beams with a circular opening strengthened by a hexagonal stiffener (HS) decreased by 5.81%, attaining 234.96 kN in comparison to the control circular beam.

A more significant decrease was noted when adopting a circular stiffener around a hexagonal opening (CS). This configuration led to the most significant reduction in structural performance among the investigated models, with the ultimate load decreasing by 11.36% to 241.50 kN. It exhibited the dataset's minimum yield displacement and maximum displacement, with values of 8.95 mm and 35.95 mm, respectively.

Displacement and ductility analysis

The ductility performance of the investigated beams was evaluated though the displacement factor, which is the ratio of the maximum displacement (Δu) to the maximum yield (Δy). The procedure for calculating the ductility factor is shown in Figure (9) and is fully defined in Equation (1).

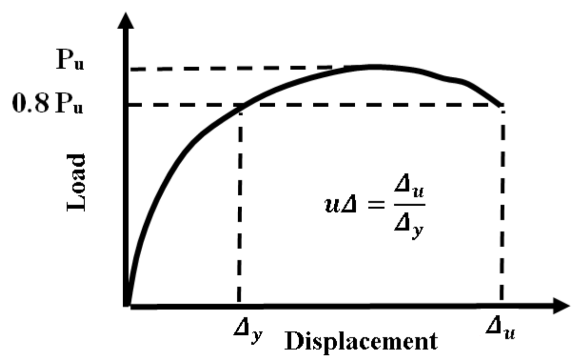

Figure 9: Definition of the ductility factor derived from the Load – Displacement response [16]

$$u\Delta = \frac{\Delta_u}{\Delta_y} \quad (1)$$

The (HEX-DH0.7-O-10-FC) beam had the highest ductility factor of 6.33 in all tested configurations. Consequently, this configuration has the highest deformation capability before failure, making it the best choice for applications requiring higher energy recovery and post-failure deformation.

Conversely, (CIR-DH0.7-O-10-FC) demonstrated the lowest ductility factor of 3.39. The flange-connected configuration, which was first used as reinforcement technique, seems to reduce the deformation capacity and to promote a relatively

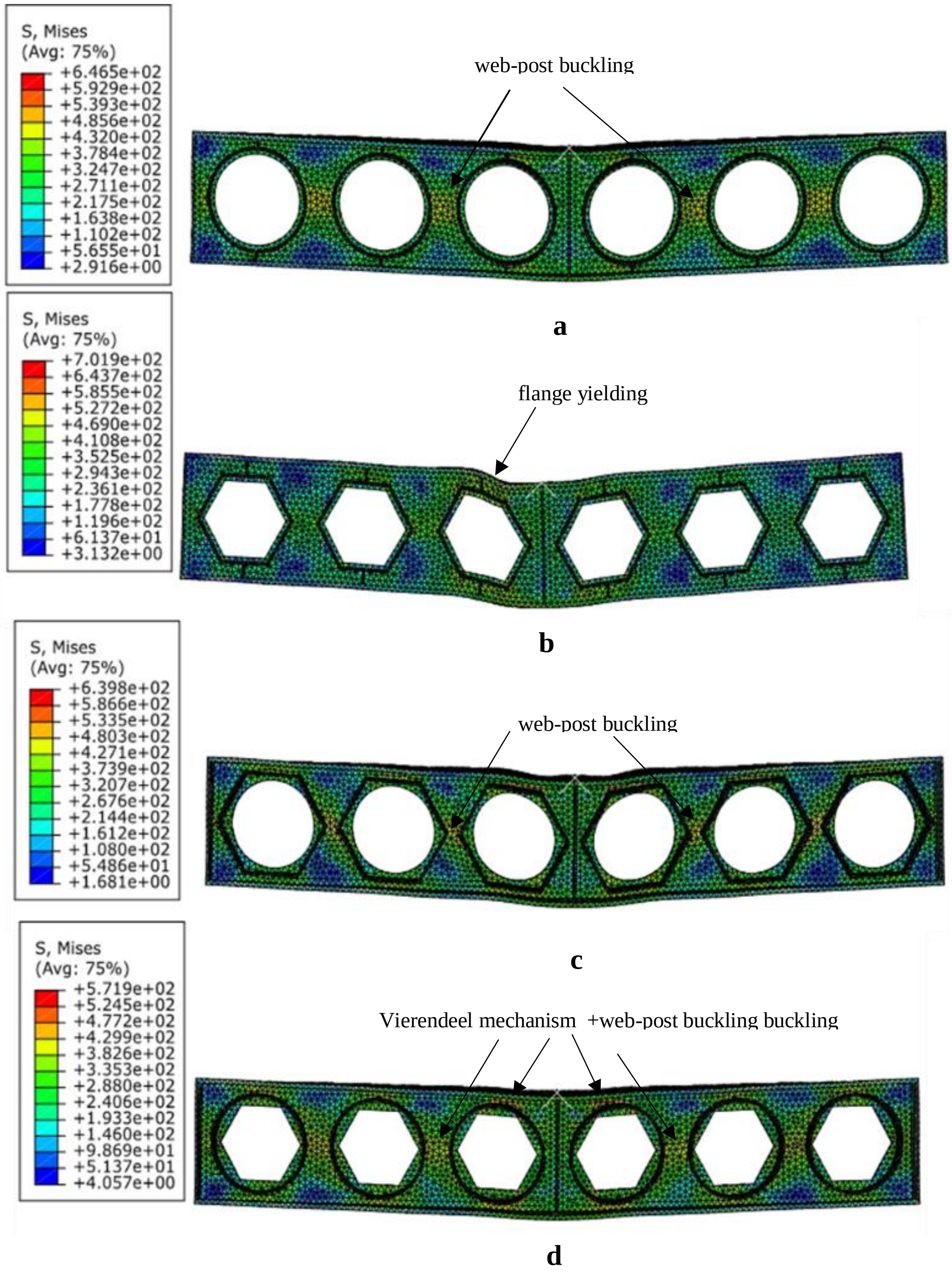

Figure 10: Failure modes: (a) CIR-DH0.7-O-10-FC -web-post buckling, (b) HEX-DH0.7-O-10-FC - flange yielding (c) CIR-DH0.7-O-10-HS- web-post buckling, and (d) HEX-DH0.7-O-10-CS -Vierendeel mechanism +web-post buckling

more brittle structural response in circular openings. Overall, with a D-H ratio of 0.7 and distance of 10 mm, the flange-connected hexagonal opening beam maximized structural performance load-carrying capacity and ductility.

Failure modes Mechanisms

Figure 10 (a) shows web post buckling induced by the concentration of compressive stresses, which reduced the beam's load-carrying capacity. Figure 10 (b) illustrates that while hexagonal openings cause increased stress concentration at the corners, this design effectively delays web buckling. It facilitates load transfer from the top flange to the bottom flange via the web, ultimately leading to flange buckling and yielding. A geometric mismatch between the openings and the ring stiffener, combined with a lack of flange contact, causes local web post buckling, as shown in Figure 10(c). The stresses remain confined within the web, and the inverted rings fail to restrain it, causing web post buckling. Figure 10(d) shows that this model exhibits a combined failure mechanism involving a Vierendeel mechanism and web post-buckling. Geometric incompatibility and the absence of a flange connection clearly contributed to this combined failure mode.

Conclusion

Based on the numerical study conducted to examine the behavior of castellated steel beams with various geometric modifications, the following key conclusions can be drawn:

• Hexagonal openings showed a clear superiority over circular openings under the same conditions. Configuration (HEX-DH0.7-O-10) achieved an ultimate load of 272.45 kN and a ductility factor of 5.65, higher than the circular openings with a load of 249.46 kN and a ductility factor of 4.25.
• The hexagonal openings connected to the flange (HEX-DH0.7-O-10-FC) achieved the highest load capacity of 289.11 kN, with an increase of 6.12% compared to the control. Meanwhile, the model showed the best deformation capacity before failure, with a ductility ratio of 6.33.
• The shape of the openings had a clear impact when connecting the ring stiffeners to the flanges. While it improved the behavior of the beams with hexagonal openings, its effect was negative on the circular beams, where the ultimate load slightly decreased, and the ductility factor reached its lowest value of 3.39 in the studied configurations.
• The inverted stiffeners had an overall negative impact on structural efficiency, since the ultimate load had decreased compared to the matched configurations. The hexagonal openings surrounded by a circular ring reinforcement showed the greatest decrease in the ultimate load, reaching 11.36%.
• The web post buckling of castellated beams featuring large hexagonal openings can be controlled by connecting the ring stiffeners to the flanges.

In summary, the best option is a beam with hexagonal openings and a hexagonal ring stiffener connected to the flanges, with a depth-to-diameter ratio of 0.7. This configuration enhances both maximum load capacity and flexibility.

Limitations of the Study

The results outlined in this study are constrained by the following limitations:
• The results are limited to a single point load at the mid-span of the beams. The results should not be generalized to other loading conditions, such as cyclic or seismic loading.
• The validity of the proposed FE model was verified with two experimental samples as reported in the literature. Generalizing the results requires conducting several experimental models covering various parameters such as (geometric shape, D/H ratio of beams, beam length, beam cross-section, and material properties).
• Despite the selected mesh size providing stable and consistent results, a comprehensive sensitivity analysis of the mesh, including multiple levels of refinement, has yet to be conducted.

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